Inverse Kinematics for a Parallel Myoelectric Elbow

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Abstract. Nowadays, myoelectric prostheses for replacement above elbow are serial mechanisms driven by a DC motor and they include only one active articulation for the elbow [1].

Parallel mechanisms are more robust and produce a greater force than serial mechanisms since every actuator participates in the desired movement of the system. Calculating the position of every actuator is more complicated than in serial mechanisms, and as a result, the mathematical models for parallel mechanisms are rather scarce [2].

The inverse kinematics model of a 3-degree of freedom parallel prosthetic elbow mechanism is reported. The mathematical model is required in order to design an above elbow myoelectric prosthesis. The prosthesis under design will have 4 active degrees of freedom and the elbow will employ a parallel mechanical system. The flexion of the elbow, the prono-supination and the humeral rotation are produced by the simultaneous participation of 3 actuators. The grasp is produced by a fourth independent motor.

Different derivations of the mathematical model will improve the design of the mechanism of the elbow, with savings in experimentation. Finally, this inverse kinematics model will be employed, using interpolation, in the first control program for the final prosthesis.

Keywords: Myoelectric prosthesis, above elbow, parallel robotics, inverse kinematics

I. Introduction

Up to date, the clinical prostheses for substitution above elbow are impelled with electric motors [1,3]. Also it has been found that each active articulation has a motor that impels only this specific articulation. So, in prostheses with three active articulations while a motor works the other two represent a load.

The section of Bioelectronics of the Center of Research and Advanced Studies of IPN of Mexico is working on a system that overcomes this problem, and trying to imitate a biological human arm have been built electromechanical muscles [4] activated by electrical motors that can work simultaneously to activate diverse articulations (see fig. 1).

While in previous systems if a pronation is required it is only necessary to activate the supinator motor until the desired angle is reached; in the system proposed at our institute [4] the coordinated participation of several motors is required. This has the benefit that motors that before were ballast now are active elements that help to carry out the required movement. But, due to the same reason it becomes necessary the inclusion of a microcontroller that coordinates the opportune participation of each motor.
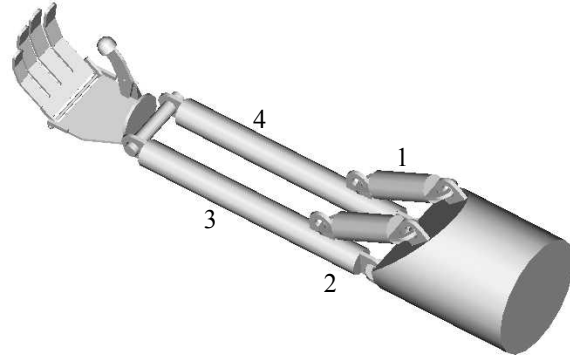

Fig. 1. Prosthetic arm with 4 actuators in parallel. This configuration allows three active degrees of freedom in the elbow, active prension of the hand and pasive flexión of the wrist.

A microprocessor will know the position of the electromechanical muscles in every moment since they are provided with positional encoders. Using the inverse kinematics model it is possible to know the position that each actuator should have to reach the final position of the forearm. Knowing the original and the ending position of the forearm it is only necessary to drive the actuators interpolating the intermediate positions. Consequently, the goal of this paper is to find the inverse kinematics model of the prosthetic arm to allow the performing of such a task.

In the following section it is made a brief description of the mechanism of the elbow.

II. Description

In figure 2 it is represented, in a single way, the schematized mechanism of figure 1. The actuator marked with number 4 in figure 1 is equivalent to the segment AH. This actuator is the responsible of driving the hand, its activation does not participate in the position of the elbow, in consequence for the elbow analysis this actuator is equivalent to a bone with a spherical joint in the proximal side. A change in the length of the actuator connected from B to I, and now referred as actuator BI, produces an equivalent to humeral rotation. The simultaneous and opposite movement of the actuators DE and CF produces the prono-supination. The simultaneous and identical movement of the same actuators DE and CF produces a flexion or extension of the elbow.

The number adjacent to each joint in figure 2 indicates the degrees of freedom for the respective joint. The electromechanical muscles are articulated with a cylindrical joint, this is, they have two degrees of freedom. In the point H there is a revolute joint that turns around u-axis. In the point A there is a spherical joint, all other joints are universal (2 degrees of freedom).

Report Documentation Page

Report Date	Report Type	Dates Covered (from... to)	
25 Oct 2001	N/A	-	

Title and Subtitle	**Contract Number**
Inverse Kinematics for a Parallel Myoelectric Elbow	
	Grant Number
	Program Element Number

Author(s)	**Project Number**
	Task Number
	Work Unit Number

Performing Organization Name(s) and Address(es)	**Performing Organization Report Number**
Center of Research and Advanced Studies of the IPN Department of Electrical Engineering Bioelectronics. Mexico, D.F. 07360	

Sponsoring/Monitoring Agency Name(s) and Address(es)	**Sponsor/Monitor's Acronym(s)**
US Army Research, Development & Standardization Group (UK) PSC 802 Box 15 FPO AE 09499-1500	
	Sponsor/Monitor's Report Number(s)

Distribution/Availability Statement
Approved for public release, distribution unlimited

Supplementary Notes
Papers from 23rd Annual International Conference of the IEEE Engineering in Medicine and Biology Society, October 25-28, 2001, held in Istanbul, Turkey. See also ADM001351 for entire conference on cd-rom.

Abstract

Subject Terms

Report Classification	**Classification of this page**
unclassified	unclassified

Classification of Abstract	**Limitation of Abstract**
unclassified	UU

Number of Pages
4

The variables for mechanism in figure 2, their description and the respective value are given next.

J=10. Number of joints in the mechanism, assuming all of them are binary.

N=8. Number of links in the mechanism, including the fixed link.

λ= 6. Degrees of freedom of the workspace

fi: Degrees of freedom of joint i.

B: Passive degrees of freedom

L: Number of independent loops in the mechanism.

According to Euler's equation the mechanism has

L= j-n +1=10-8+1=3

This is, there are three independent loops. Employing the mobility criterion for loops in parallel mechanisms we have

$$B = \sum fi - F - \lambda L$$
$$B = 22 - 3 - 6(3) = 1$$

This means that the system has three active degrees of freedom and one passive (remember that the active grasping is not being considered in this analysis)

The three degrees of freedom belong to flexion-extension of elbow, prono-supination and humeral rotation. The passive degree of freedom belongs to flexion-extension of the wrist.

According to the last analysis it is not possible to replace a joint using another one with fewer degrees of freedom without limiting the movement of the prosthesis. For example, in points J, K and L ,it is not possible to replace the cylindrical joints, belonging to the actuators, with prismatic joints.

III. Homogeneous Transformation Matrices

In figure 2, there is a coordinate system xyz in point A, which is fixed to the base of the prosthetic arm, it will be called the fixed system. Axes of the fixed system are as shown in figure 2. In the other side, in point G there is a coordinate system uvw that moves jointly with link HI, this will be called as the mobile system. Axes in the mobile system have the next orientation. The origin is always on G, this is, at mid range on the line HI. The u-axis is parallel to line HI and its direction is from H toward I. The w-axis is parallel to AH as shown. The v-axis is orthogonal to the other two axes and its direction obeys the right hand rule.

If the orientation and distance between both coordinate systems are known, it is possible to build a transformation matrix that get the coordinates referred to the fixed system of a point situated on the moving system.

A homogeneous transformation matrix is a 4x4 matrix defined to transform a homogeneous position vector from the coordinate system B to the coordinate system A as follows.

$$^A p = {^A T_B}{^B p}$$

Where

$$^A T_B = \begin{bmatrix} ^A R_B & \vdots & ^A q \\ \cdots & \vdots & \cdots \\ 000 & \vdots & 1 \end{bmatrix}$$

$$^A p = \left[p_x, p_y, p_z, 1 \right]^T$$

$$^B p = \left[p_u, p_v, p_w, 1 \right]^T$$

The transformation matrix is subdivided in four submatrices, the 3x3 submatrix at left and up $^A R_B$ represents the orientation of a mobile coordinate system B with reference to a fixed coordinate system A. The 3x1 submatrix at right, Aq, indicates the origin of the mobile coordinate system in relation to the fixed one. The 1x3 submatrix at down and left, γ, represents a perspective transformation and the submatrix 1x1 down and right, known as ρ, is the scale factor. For this kinematic analysis the perspective matrix γ is set to zero and the scale factor matrix ρ is set to 1.
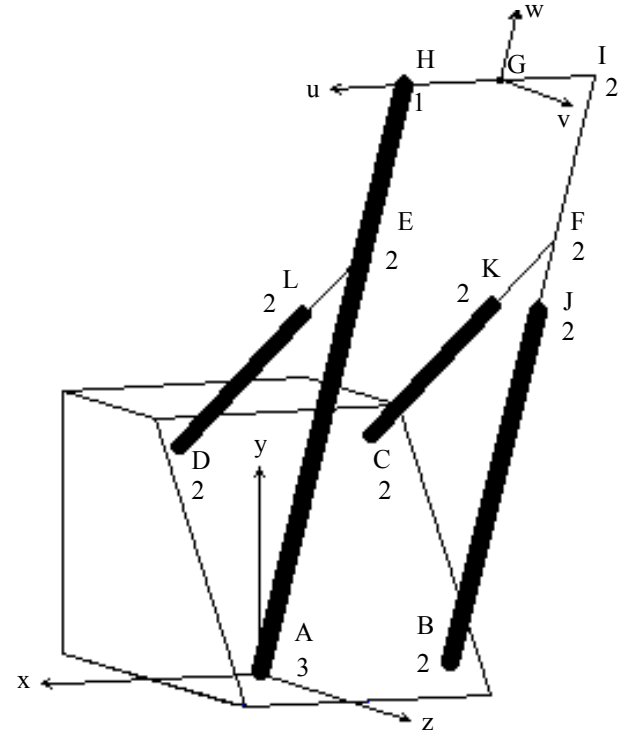

Fig. 2. Schematic model of the prosthetic elbow simplified to allow the development of the inverse kinematics model of the elbow. The links, joints, degrees of freedom of each joint, actuators and fixed and mobile coordinate systems are shown.

As an example, the transformation matrix for a simple rotation about the z-axis is given by

$$^{A}T_{B}(z,\theta) = \begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and the matrix for a pure translation is given by

$$^{A}T_{B}(q) = \begin{bmatrix} 1 & 0 & 0 & q_{x} \\ 0 & 1 & 0 & q_{y} \\ 0 & 0 & 1 & q_{z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The sequence of the movements of the elbow is not commutable as the matrix multiplication is not commutable. As a result, the multiplication must be performed in the same order as movements are realized as long as the reference system is the fixed system [5].

IV. The transformation matrix

In order to estimate the transformation matrix that relates the mobile coordinate system in G with the fixed coordinate system in A, there are proposed the following steps:

First it is assumed that both systems are originally coincident in position and orientation and the mobile coordinate system will be translated and rotated until it reaches the position on G and the required orientation.

The mobile system is rotated an angle α about the x or u-axis (in this moment both axes are coincident), necessary to match the w-axis with the elevation of link AH. Second, It is performed a new rotation of ϕ about the new v-axis in order to match completely the w-axis with the link AH. Third, a rotation of θ about the new w-axis is realized in order to align the u-axis completely parallel with the link HI. In this way, the mobile system has the final orientation, now it is necessary to displace the frame to the final position. For this, we multiply by a translation matrix that involves a displacement d along the new w-axis and a displacement a along the u-axis.

The equation that performs all these rotations and translations is the next:

$$^{G}A_{A} = R(x,\alpha) \cdot R(v,\phi) \cdot R(w,\theta) \cdot T(w,d) \cdot T(u,a)$$

Where R represents a rotation matrix and T represents a translation matrix. The result of this multiplication is the next:

$$^{G}A_{A} = \begin{bmatrix} c\theta c\phi & -s\theta c\phi & s\phi & ds\phi - ac\theta c\phi \\ c\alpha s\theta + s\alpha c\theta s\phi & c\alpha c\theta - s\alpha s\theta s\phi & -s\alpha c\phi & -a(c\alpha s\theta + s\alpha c\theta s\phi) - ds\alpha c\phi \\ s\alpha s\theta + s\alpha c\theta s\phi & c\alpha s\theta s\phi + s\alpha c\theta & c\alpha c\phi & a(c\alpha c\theta s\phi - s\alpha s\theta) + dc\alpha c\phi \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

V. Computing the length of the actuators. Inverse Kinematics

Using the transformation matrix $^{G}A_{A}$ it can be computed the equivalent coordinates in the fixed system xyz of any point in the mobile system uvw, for example the point H in figure 2 has the coordinates $H_{uvw}=[a\ 0\ 0\ 1]^{T}$ then the same point has the equivalent coordinates

$$H_{xyz} = {}^{G}A_{A}H_{uvw}$$

and if $\alpha = 0$, $\phi = 0$ and $\theta = 0$, then

Hxyz=[0 0 d 1]^{T}

These are the coordinates of the same point H seen from the fixed frame in A.

Now we have the tools for computing the length of the actuators. To compute the actuator BI length the procedure is as follows. First, compute the coordinates of the point I respect to fixed system xyz.

Ixyz=^{G}A$_{A}$Iuvw

obtaining

Ixyz=[I$_{x}$ I$_{y}$ I$_{z}$ 1]^{T}

Point B is fixed to the xyz system and it has the next coordinates

Bxyz=[b$_{x}$ b$_{y}$ b$_{z}$ 1]

The actuator BI has a length

$$|BI| = \sqrt{\left(B_{xyz} - I_{xyz}\right)^{2}}$$

The length of actuator DE will be computed in the same way

E has the next coordinates

Euvw=[a 0 e$_{w}$ 1]

Exyz= ^{G}A$_{A}$Euvw

Exyz=[e$_x$ e$_y$ e$_z$ 1]^T

D has fixed coordinates with respect to xyz and does not require any transformation. The length of the actuator DE is then

$$|DE| = \sqrt{\left(D_{xyz} - E_{xyz}\right)^2}$$

To compute the length of the actuator FC is more complicated

First it is necessary to estimate the unitary vector *IBu* which is parallel to IB

$$IBu = \frac{IB}{|IB|}$$

The point F has the next coordinates

$$F_{uvw} = I_{uvw} + |IF|\,IBu$$

Where $|IF|$ is the distance from I to F, where this distance is constant.

F$_{xyz}$=^{G}A$_A$F$_{uvw}$

And finally the distance FC is

$$|FC| = \sqrt{\left(F_{xyz} - C_{xyz}\right)^2}$$

where C$_{xyz}$ is fixed with respect to xyz and its coordinates are known.

Now we know the length of the three actuators, BI, DE and FC that determine the position of the prosthetic elbow given their angles α, ϕ and θ. This concludes the inverse kinematics model.

VI. Conclusion

A kinematic model for a prosthetic elbow has been developed. This is important because now, the length of the actuators can be known given the flexion, humeral rotation and pronosupination angles of the elbow and a microprocessor can control the actuators based on this information.

We have verified the efficacy of the model with a Matlab program that performs the simulation of the mechanism of the elbow. This kinematic model together with the Matlab simulation program has helped to find the maximum and minimum length as well as the diameter of the actuators.

The position of the insertion of the actuators in the prosthetic arm has been estimated using the kinematic model together with the static model.

In the final version of the prosthesis, a microprocessor will control the movements of the prosthesis based on the kinematic model. Once the microprocessor knows the starting position of the elbow and is instructed with the final position information through myoelectric electrodes [6], it will compute the intermediate positions to drive the actuators.

The homogeneous transformation matrix method widely used in serial robotics has been successfully used to resolve the kinetic model of a parallel prosthetic elbow mechanism.

VII. References

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